

(SAMPLE PAPER-MATHEMATICS)
(CLASS- XII)

MM:100
Duration:-3hrs

SECTION A

1. Find the value of 2×2 matrix A if, $A \cdot \begin{pmatrix} 5 & 4 \\ 1 & 1 \end{pmatrix} = \begin{pmatrix} 1 & -2 \\ 1 & 3 \end{pmatrix}$

OR

Solve for X and Y:

$$2X+3Y=\begin{pmatrix} 4 & 6 \\ 2 & -3 \end{pmatrix} \quad \& \quad 3X+7Y=\begin{pmatrix} -2 & 1 \\ 5 & -4 \end{pmatrix}$$

2. Using properties of determinants prove that:

$$\begin{vmatrix} 1+a & 1 & 1 \\ 1 & 1+b & 1 \\ 1 & 1 & 1+c \end{vmatrix} = abc(1/a+1/b+1/c+1)$$

3. In a family the husband tells a lie in 30% cases and wife in 35% cases. Find the probability that both contradict each other on the same fact.

4. A and B toss a coin alternately till one of them tosses a head and wins the game. A starts the game, find their respective chances of winning.

5. Evaluate $\int (2x+3)\sqrt{(2x^2+5x-6)}dx$.

6. Evaluate $\int \frac{x+1}{(x+2)^2} e^x dx$

7. Solve the differential equation: $x \log x \frac{dy}{dx} + y = 2/x (\log x)$.

8. Solve the differential equation: $x^2 y dx - (x^3 + y^3) dy = 0$.

9. Simplify the Boolean expression: (i) $(x+y) + (x'.y')$ (ii) $(x+y). (x'.y')$

10. Find $\lim_{x \rightarrow 1} (x-1) \tan(\pi x / 2)$

11. If $y = e^x \tan^{-1} x$, Prove that $(1+x^2) y(2) - 2(1-x+x^2) y(1) + (1-x)^2 y = 0$.

12. verify Roll's theorem for $f(x) = \sin x - \sin 2x$ in $(0, 2\pi)$.

13. Find the points on the curve $y = x^3 - 2x^2 - 2x$ at which the tangent lines are parallel to $y = 2x - 3$.

OR

Find the intervals in which the function $f(x) = x^3 - 3x^2 - 24x + 5$ is increasing or decreasing.

14. Evaluate : $\int \frac{x \, dx}{(x+1)(x^2+1)}$

15. Evaluate : $\int_0^{\pi/4} \frac{(\sin x + \cos x) \, dx}{(9+16 \sin 2x)}$

16. Solve the following system of equations

$$\frac{2}{x} + \frac{3}{y} + \frac{10}{z} = 4$$

$$\frac{4}{x} - \frac{6}{y} + \frac{5}{z} = 1$$

$$\frac{6}{x} + \frac{9}{y} - \frac{20}{z} = 2$$

17. A square piece of tin of side 24 cm is to be made into a box without top by cutting a square from each corner and folding up the flaps to form a box. What should be the side to be cut off so that the volume of the box is maximum. Also find maximum volume.

OR

Show that volume of greatest cylinder which can be inscribed in a cone of height h and semi vertical angle α is $\frac{4}{27} \pi h^3 \tan^2 \alpha$.

18. Sketch the region enclosed between the circles $x^2 + y^2 = 4$ & $(x-2)^2 + y^2 = 4$.

OR

Using integration find the area of triangle whose vertices are $(-1,1)$; $(0,5)$; $(3,2)$.

SECTION C

19. A bill drawn on 4th Feb. 1999 at four months after date was discounted on 26th March 1999 at 4% per annum. If the bank discount is Rs. 100, find the amount for which bill was drawn.

20. The difference between true and banker's discount on a bill due after 3 months at 5% interest is Rs. 100. Find (1) True Discount (2) The face value of the bill.

21. There are 6% defective items in a large bulk of items. Find the probability that a sample of 8 items will include not more than one defective item.

OR

Find the probability that no defective fuse will be found in a box of 200 fuses if experience shows that 2% of such fuses are defective [$e^{-4} = 0.018$].

22. A man buys a computer for Rs 70000 on the following condition. He will pay Rs. 10000 cash down and the balance in 10 equal quarterly installments. If the first is to be paid three months after the date of purchase, calculate the amount of each installment, interest being calculated at the rate of 5% p.a. compounded quarterly.

23. In an examination, an examinee either guesses or copies or knows the answer of multiple choice questions with four choices. The probability that he makes a guess is $\frac{1}{3}$ and probability that he copies answer is $\frac{1}{6}$. The probability that his answer is correct, given that he copied it is $\frac{1}{8}$. Find the probability that he knew the answer, given that he correctly answered it.

24. X, Y, Z are partners in a business with capitals of Rs. 5000, Rs 6000 and Rs 4000. X gets 20% of profit for management and the balance is divided in proportion to their capitals. At the end of the year X gets Rs 200 less than Y and Z together. Find the total profit and share of each.

25. A steel plant is capable of producing x tones of a low grade steel and y tones of high grade steel, where $y = \frac{5x - 40}{x - 10}$. If the fixed market price of low grade steel is one-third of that of high grade steel, find the level of output of low grade steel per day for maximum revenue.

OR

A travel agent arranges a tour from Delhi to Shimla and back. He has 60 seats in a special bus at booking amount of Rs 450 per seat provided all seats will be occupied. However for every increase of Rs 15 in the booking amount one seat will remain vacant. He also plans to provide a mineral water and snacks costing Rs 60 per seat. Find the relationship between the profit and number of seats remained vacant. What is the number of vacant seats for which profit is maximum.

26. A company sells two different products A, B. Both are produced in a common production process, which has a total capacity of 500 man hours. It takes 5 hours to produce a unit of A and 3 hours to produce a unit of B. The market has been surveyed and the company officials felt that the maximum number of units of A that can be sold is 70 and that for B is 125. If the profit is Rs 20 per unit of A and Rs 15 per unit for B, how many units of each product should be sold to maximize profit.

OR

A company produces two types of belts A and B. profits on these types are Rs 2 and Rs. 1.5 on each, belt, respectively. A belt of type A requires twice as much time as a belt of type B. The company can produce at the most 1000 belts of type B per day. Material for 800 belts per day is available At the most 400 buckles for belts of type A and 700 for those of type B are available per day. How many belts of each type should the company produce so as to maximize the profit?
