CRASH COURSE

MATHEMATICS (Assignment – 20)

[Differential Calculus – II (Differential & Higher Derivatives)]

1. If f(x) = ax + b and g(x) = cx + d, then f(g(x)) = g(f(x)) if
   (a) f(a) = g(c) (b) f(b) = g(a) (c) f(d) = g(b) (d) f(c) = g(a)

2. If values of b and c for which the identity f(x + 1) – f(x) = 8x + 3 is satisfied, where $f(x) = bx^2 + cx + d$, are
   (a) b = 2, c = 1 (b) b = 4, c = -1 (c) b = -1, c = 4 (d) b = -1, c = 1

3. If y = 2x + arc cot x + In $\left\{\sqrt{1+x^2} - x\right\}$, then y
   (a) increases in [0, ∞) (b) decreases in [0, ∞)
   (c) neither increases nor decreases in [0, ∞) (d) increases in (-∞, ∞)

4. Let f(x) = sin x + cos x be defined in [0, 2π], then f(x)
   (a) increases in $\left[\frac{\pi}{4}, \frac{\pi}{2}\right]$ (b) decreases in $\left[\frac{\pi}{4}, \frac{5\pi}{4}\right]$
   (c) increases in $\left[0, \frac{\pi}{4}\right] \cup \left[\frac{5\pi}{4}, 2\pi\right]$ (d) decreases in $\left[0, \frac{\pi}{4}\right] \cup \left[\frac{\pi}{2}, 2\pi\right]$

5. Let x be any real number, then [x + y] = [x] + [y] holds for
   (a) $y \in R$ (d) $y \in I$ (c) $y \in Q$ (d) $y \in R, y \notin Q$

6. Let g(x) be a function defined in [-1, 1]. If the area of the equilateral triangle with two of its vertices at (0, 0) and (x, g(x)) is √3/4, then
   (a) $g(x) = \pm\sqrt{1-x^2}$ (b) $g(x) = -\sqrt{1-x^2}$ or $g(x) = \sqrt{1-x^2}$
   (c) $g(x) = \sqrt{1-x^2}$ (d) $g(x) = -\sqrt{1-x^2}$

7. The maximum possible domain and corresponding range for the real function $f(x) = (-1)^x$ to exist is
   (a) $D_f = R, R_f = [-1, 1]$ (b) $D_f = I, R_f = \{1, -1\}$
   (c) $D_f = I, R_f = [-1, 1]$ (d) $D_f = R, R_f = \{-1, 1\}$

8. If $y = ce^{x/(x - a)}$, then dy/dx equals
   (a) $a(x – a)^2$ (b) $–ay/(x – a)^2$ (c) $a^2(x – a)^2$ (d) None of these

9. If $e^y + xy = e$, then the value of d^2y/dx^2 for x = 0 is
   (a) 1/e (b) $1/e^2$ (c) $1/e^3$ (d) None of these

10. Let f be a twice differentiable function such that f''(x) = -f(x) and f'(x) = g(x). If $h(x) = [f(x)]^2 + [g(x)]^2$, h(1) = 8, then h(2) is equal to
    (a) 1 (b) 2 (c) 3 (d) None of these

11. The value of y''(1) if $x^3 – 2x^2y^2 + 5x + y – 5 = 0$ when y(1) = 1, is equal to
    (a) 22/7 (b) –7(21/28) (c) 8 (d) –8(22/27)

12. If $x = \sin^{-1}t$ and $y = \log(1 – t^2)$, then $\left.\frac{d^2y}{dx^2}\right|_{t=1/2}$ is
    (a) –3/4 (b) 3/2 (c) –8/3 (d) None of these

13. If f is twice differentiable function such that f''(x) = -f(x), and f'(x) = g(x), $h(x) = [f(x)]^3 + [g(x)]^3$ and h(5) = 11 then h(10) is equal to
(a) 7 (b) $-2x/(1+x^4)$ (c) $-1/(1+ x^4)$ (d) None of these

14. If y^2 = P(x) is a polynomial of degree 3, then $2\frac{d}{dx}\left\{y^3\frac{d^2y}{dx^2}\right\}$ equals
(a) P'''(x) + p'(x) (b) P''(x) + P'''(x) (c) P(x).P'''(x) (d) a constant

15. Let y = ln $(x^2 + x - 2)$, then y_n is equal to
(a) $(n-1)!\left[\frac{1}{(x-1)^n}+\frac{1}{(x+2)^n}\right]$ (b) $(-1)^n.(n-1)!\left[\frac{1}{(x-1)^n}+\frac{1}{(x+2)^n}\right]$
(c) $(-1)^{n-1}.(n-1)!\left[\frac{1}{(x-1)^n}+\frac{1}{(x+2)^n}\right]$ (d) None of these.

16. If y = x sin x, then $(y_{10})_0$ is equal to
(a) 10 (b) –10 (c) (10)! (d) –(10)!

17. If y = $\tan^{-1} x)^2$, then $(x^2 + 1)^2 y_2 + 2x(x^2 + 1) y_1$ is equal to
(a) 2 (b) –2 (c) 2y (d) –2y

18. Let f(x) = sin x, g(x) = x^2 and h(x) = $\log_e x$. If F(x) = (hogof)x, then F''(x) is equal to
(a) 2 $\text{cosec}^3 x$ (b) 2 cot $x^2 - 4x^2$ $\text{cosec}^2\, x^2$ (c)2x $\cot^2 x$ (d) –2 $\text{cosec}^2 x$

19. If y = $\sin^{-1}\sqrt{(1-x)} + \cos^{-1}\sqrt{x}$, then dy/dx is equal to
(a) $\frac{1}{\sqrt{[x(1-x)]}}$ (b) $\frac{-1}{\sqrt{[x(1-x)]}}$ (c) $\frac{1}{\sqrt{[x(1+x)]}}$ (d) None of these

20. Let y = $\cos^{-1}[(2\cos x + 3\sin x)/\sqrt{(13)}]$. Then $\frac{dy}{dx} = 1$
(a) for all x (b) for $\tan^{-1} 3/2 < x < \pi + \tan^{-1} 3/2$
(c) for $-\{\pi - \tan^{-1}(3/2)\} < x < \tan^{-1}(3/2)$ (d) for no x.

21. Let y = $\tan^{-1}\left(\frac{4x}{1+5x^2}\right)+\tan^{-1}\left(\frac{2+3x}{3-2x}\right)$. Then $\frac{dy}{dx}$ is equal to
(a) $1/(1 + 25x^2)$ (b) $5/(1 + 25x^2)$ (c) $5/\sqrt{(1 + 25x^2)}$ (d) None of these

22. If f(x) = $\tan^{-1}\sqrt{[(1 + \sin x)/(1 - \sin x)]}$, $0 \le x \le \pi/2$, the f'(π/6) is equal to
(a) –1/4 (b) –1/2 (c) 1/4 (d) 1/2

23. If sin (x + y) = log (x + y), then dy/dx is equal to
(a) 2 (b) –2 (c) 1 (d) –1

24. The derivative of the function $\cot^{-1}[(\cos 2x)^{1/2}]$ at x = π/6 is
(a) $(2/3)^{1/2}$ (b) $(1/3)^{1/2}$ (c) $3^{1/2}$ (d) $6^{1/2}$

25. If $x^y = e^{x-y}$, then dy/dx is equal to
(a) $\frac{y}{1+\log x}$ (b) $\frac{x-y}{(1+\log x)^2}$ (c) $\frac{x-y}{1+\log x}$ (d) $\frac{\log x}{(1+\log x)^2}$

26. If y = √[sin x + √{sin x + √(sin x + ...)}], then dy/dx is equal to

(a) $\dfrac{2y-1}{\cos x}$ (b) $\dfrac{\cos x}{2y-1}$ (c) $\dfrac{2x-1}{\cos y}$ (d) $\dfrac{\cos y}{2x-1}$

27. If y = $\tan^{-1}$(sec x – tan x), then dy/dx is equal to

(a) 2 (b) –2 (c) 1/2 (d) –1/2

28. If $\Delta_1 = \begin{vmatrix} x & a & a \\ b & x & a \\ b & b & x \end{vmatrix}$ and $\Delta_2 = \begin{vmatrix} x & a \\ b & x \end{vmatrix}$, then

(a) $\dfrac{d}{dx}\Delta_1 = \Delta_2$ (b) $\dfrac{d}{dx}\Delta_1 = 3\Delta_2$ (c) $\dfrac{d}{dx}\Delta_2 = \Delta_1 - \Delta_2$ (d) None of these

29. Given : $2x^2 - xy - y^2 = 0$. Then $\left[\dfrac{dy}{dx}\right]_{(1,-2)}$ is equal to

(a) –2 (b) –3/2 (c) 4/3 (d) 2

30. If y = $\log_{\sin x}$ (tan x), then $(dy/dx)_{\pi/4}$

(a) 4/log 2 (b) –4 log 2 (c) –4/(log 2) (d) None of these

31. y = $\tan^{-1}\left(\dfrac{\log(e/x^2)}{\log(ex^2)}\right) + \tan^{-1}\left(\dfrac{3+2\log x}{1-6\log x}\right)$, then y” is equal to

(a) 1/2 (b) 2 (c) –1 (d) None of these

32. If y = a(1 + cos θ), x = a(θ + sin θ), then $\dfrac{d^2y}{dx^2}$ at θ = π/2 is

(a) –1/a (b) –1 (c) 0 (d) None of these

33. If x = a $\cos^3$ θ, y = b $\sin^3$ θ, then y_2 at θ = 0 is

(a) –b/a (b) 0 (c) ∞ (d) Not exist

34. If f(x) = |x – 2| and g(x) = f(f(x)), then g’(4) equals

(a) –1 (b) 1 (c) 0 (d) None of these

35. The derivatives of $\sin^{-1}\left(\dfrac{\sqrt{1+x}+\sqrt{1-x}}{2}\right)$ with respect to x is

(a) $-\dfrac{1}{2\sqrt{1-x^2}}$ (b) $\dfrac{1}{2\sqrt{1-x^2}}$ (c) $\dfrac{2}{2\sqrt{1-x^2}}$ (d) $\dfrac{-2}{2\sqrt{1-x^2}}$

36. If f(x) = $\begin{vmatrix} \sec\theta & \tan^2\theta & 1 \\ \theta \sec x & \tan x & x \\ 1 & \tan x - \tan\theta & 0 \end{vmatrix}$, then f’(θ) is

(a) 0 (b) –1 (c) independent of θ (d) None of these

37. If x = ϕ(t), y = Ψ(t), then d^2y/dt^2 is equal to

(a) $\frac{\phi'\psi''-\psi'\phi''}{(\phi)^2}$ (b) $\frac{\phi'\psi''-\psi'\phi''}{(\phi')^3}$ (c) $\frac{\phi''}{\psi''}$ (d) $\frac{\psi''}{\phi''}$

38. Let f(x + y) = f(x)f(y) for all x and y. Suppose f(5) = 2 and f'(0) = 3, then f'(5) is equal to

(a) 4 (b) 6 (c) 12 (d) None of these

39. Let f(x) = x for $x \le 1$ and f(x) = $2x - x^2$ for x > 1. Then f'(x) is continuous

(a) every where (b) at x = 1

(c) every where except x = 1 (d) None of these

40. Let g(x) = $\frac{1}{2}x^2$ for $0 \le x \le 1$ and g(x) = $2x^2 - 3x + \frac{3}{2}$ for $x \ge 1$. The g''(x) is continuous

(a) every where (b) at x = 1

(c) every where except x = 1 (d) None of these

41. Let p(x) = $\begin{vmatrix} x & x^2 & x^3 \\ 1 & 2x & 3x^2 \\ 0 & 2 & 6x \end{vmatrix}$. Then p'(2) is equal to

(a) 0 (b) 6 (c) 24 (d) None of these

42. If f(x) = $\cos(x^2 - 2[x])$ for 0 < x < 1, then $f'\left(\sqrt{\frac{\pi}{2}}\right)$ is equal to

(a) $-\sqrt{\frac{\pi}{2}}$ (b) $\sqrt{\frac{\pi}{2}}$ (c) 0 (d) None of these

43. If f(x) = $\log_{x^2}(\ln x)$, then f'(x) at x = e, is

(a) 0 (b) 1 (c) 1/e (d) 1/(2e)

44. If Q(x) is inverse of P(x) and $(1 + x^n)$P'(x) = 1, then Q'(x) is equal to

(a) $1 + [Q(x)]^n$ (b) $1 + [P(x)]^n$ (c) $1 + [Q(x)]^{-n}$ (d) None of these

45. If y = $\tan^{-1}[(1 - x^2)/(1 + x^2)]$, then dy/dx is equal to

(a) $1/(1 + x^4)$ (b) $1 + [P(x)]^n$ (c) $1 + [Q(x)]^{-n}$ (d) None of these

46. The differential coefficient of f(log x) w.r.t. x, where f(x) = log x is equal to

(a) x/log x (b) log x/x (c) $-1/(1 + x^4)$ (d) None of these

47. If y = $(1 + 1/x)^x$ then dy/dx is equal to

(a) $\left(1+\frac{1}{x}\right)^x\left[\log\left(1+\frac{1}{x}\right)-\frac{1}{1+x}\right]$ (b) $\left(1+\frac{1}{x}\right)^x\left[\log\left(1+\frac{1}{x}\right)\right]$

(c) $\left(1+\frac{1}{x}\right)^x\left[\log(x+1)-\frac{x}{1+x}\right]$ (d) $\left(1+\frac{1}{x}\right)^x\left[\log\left(1+\frac{1}{x}\right)+\frac{1}{1+x}\right]$

48. If y = log|x|, $x \ne 0$, then dy/dx is given by

(a) 1/|x| (b) 1/x (c) $-1/x$ (d) does not exist

49. If $x^y = y^x$, then dy/dx is

(a) $\frac{y(y+x\log y)}{x(y\log x+x)}$ (b) $\frac{y(x+y\log x)}{x(y+x\log y)}$ (c) $-\frac{y(y+x\log y)}{x(x+y\log x)}$ (d) None of these

50. If y = $\sec^{-1}\frac{2x}{1+x^2} + \sin^{-1}\frac{x-1}{x+1}$, 0 < x < 1, then $\frac{dy}{dx}$ is

(a) 1 (b) $\frac{x-1}{x+1}$ (c) 0 (d) None of these

51. Differential Coefficient of $\sec(\tan^{-1}x)$ is

(a) $x/(1 + x^2)$ (b) $x\sqrt{(1 + x^2)}$ (c) $1/\sqrt{(1 + x^2)}$ (d) $x/\sqrt{(1 + x^2)}$

52. If y = $\sec^{-1}\left(\frac{x+1}{x-1}\right)+\sin^{-1}\left(\frac{x-1}{x+1}\right)$, then dy/dx is equal to

(a) 1 (b) 0 (c) (x – 1)/(x + 1) (d) (x + 1)/(x – 1)

53. f(x) = $\frac{\sqrt{1+px}-\sqrt{1-px}}{x}$, $-1 \le x < 0$; f(x) = $\frac{2x+1}{x-2}$, $0 \le x \le 1$ is continuous in the interval [-1, 1], then p is equal to

(a) –1 (b) –1/2 (c) 1/2 (d) 1

54. If $b^2 - 4ac = 0$, a > 0, then the domain of the function
y = $\log(ax^3 + (a + b)x^2 + (b + c)x + c)$ is

(a) R - $\left\{-\frac{b}{2a}\right\}$ (b) R - $\left\{\left\{-\frac{b}{2a}\right\}\cup(x : x \ge -1\right\}$

(c) R - $\left\{\left\{-\frac{b}{2a}\right\}\cap(-\infty,-1\right\}$ (d) none of these

55. If (x) = $\frac{1-x}{1+x}$, then, the domain of $f^{-1}(x)$ contain

(a) (-∞, ∞) - {-1} (b) (-∞, -1) (c) (1, ∞) (d) none of these

56. The domain of definition of the function $\sqrt[3]{\frac{2x-1}{x^2-10x-11}}$ is given by

(a) -∞ < x < ∞ (b) x > 0 (c) x < 0 (d) x ≠ -1, x ≠ 11

57. If D is the set of all real x such that 1 - $e^{[1/x]-1}$ is positive, then D is equal to

(a) (-∞, 1) (b) (-∞, 0) (c) (1, ∞) (d) (-∞, 0) ∪ (1, ∞)

58. The range of the function f(x) = $\frac{1-x^2}{x^2}$ is equal to

(a) [0, 1] (b) (0, 1) (c) (1, ∞) (d) [1, ∞)